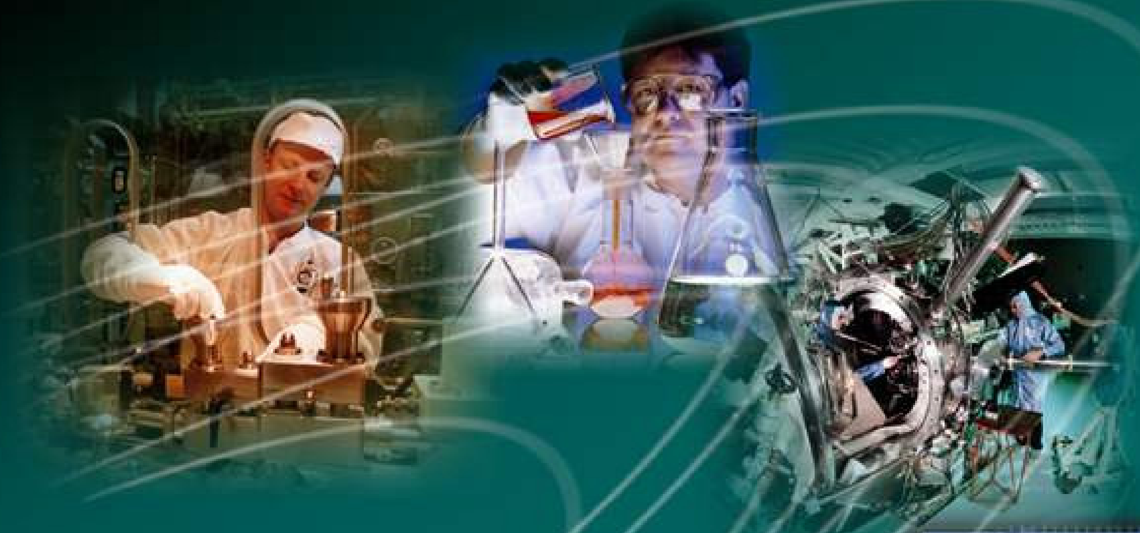
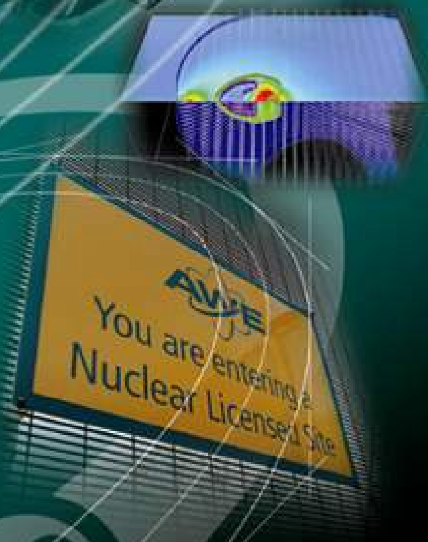
Compatible Finite Element multi-material Hydro





Oxford
September 5-8th

Andrew Barlow
AWE

Introduction (1)

- The Finite volume compatible Lagrangian Hydro methods developed by Caramana, Shashkov, Burton et al potentially offers significant improvements over traditional PdV based schemes.
- A new compatible Finite Element Lagrangian hydro scheme has been developed for CORVUS based on this approach.
- The new method was developed principally to assess the benefits that compatible hydro may offer over the existing PdV based Lagrangian Hydro scheme used in CORVUS.

Introduction (2)

- But also to test the generality of the compatible hydro approach and see if the ideas can be translated across from finite volume to other types of numerical methods.
- It was also considered desirable to retain as much as possible of the original well validated and well understood Finite Element Hydro scheme currently used in CORVUS.

Compatibility 1

- Compatibility simply refers to making the internal energy or work update consistent with the momentum update.
- This is achieved by replacing the usual PdV internal energy update by:

$$M_z \frac{de_z}{dt} = -\sum_p \vec{f}_p^{\,z} . \vec{v}_p$$

- where f are the forces calculated during the momentum step.
- The corner masses must also be treated as Lagrangian objects just like the zone masses.

Compatibility 2

- Compatible hydro has two main things to offer:
  - Total energy conservation to round off for the Lagrangian step.
  - Increased flexibilty in types of forces that can be allowed within a zone (e.g. sub-zonal pressures and edge q's).

Cylindrical Geometry

- Since the original CORVUS hydro scheme uses Petrov Galerkin (or area weighting) this has been retained to allow the force calculation for the momentum step to remain unchanged.
- In order to use these area weighted forces the internal energy update has to be modified to:

$$M_z \frac{de_z}{dt} = -\sum_p r_p \vec{f}_p^{\,z} . \vec{v}_p$$

Corner mass definition 1

- In order for this form for the internal energy update to be valid, the corner mass must also be related to the area weighted corner masses (areal inertia) in the same way as the area weighted forces used in the momentum are related to the real forces.

$$(\rho A)_p = \frac{M_p}{r_p}$$

- This means the current finite element approach for defining the area weighted and real nodal masses must be abandoned.

Corner mass definition 2

- The area weighted corner masses (areal inertia) are found by requiring the momentum equation to take the form:

$$M_p \frac{d\vec{v}_p}{dt} = r_p (\rho A)_p \frac{d\vec{v}_p}{dt} = r_p \sum_z \vec{f}_z^{\,p}$$

- The areas can be calculated as suggested by Caramana et al by subdividing the zone into 4 triangles each with vertices given by the end points of a zone edge and the center of the zone $r_c=(r_1+r_2+r_3+r_4)/4$ and noting the true volume of each of these triangles is $A_i(r_i+r_{ri+1}+r_c)/3$
- The volumes of these triangles will then sum to give the total cell volume. This can then be decomposed with respect to r_i to define the new corner volumes and area weights.

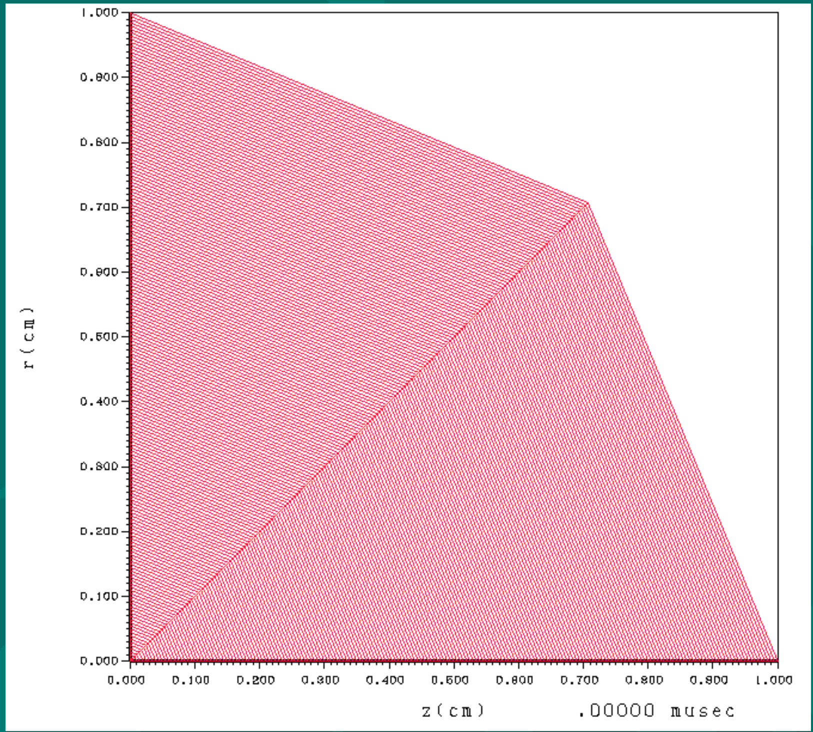
Corner mass definition 3

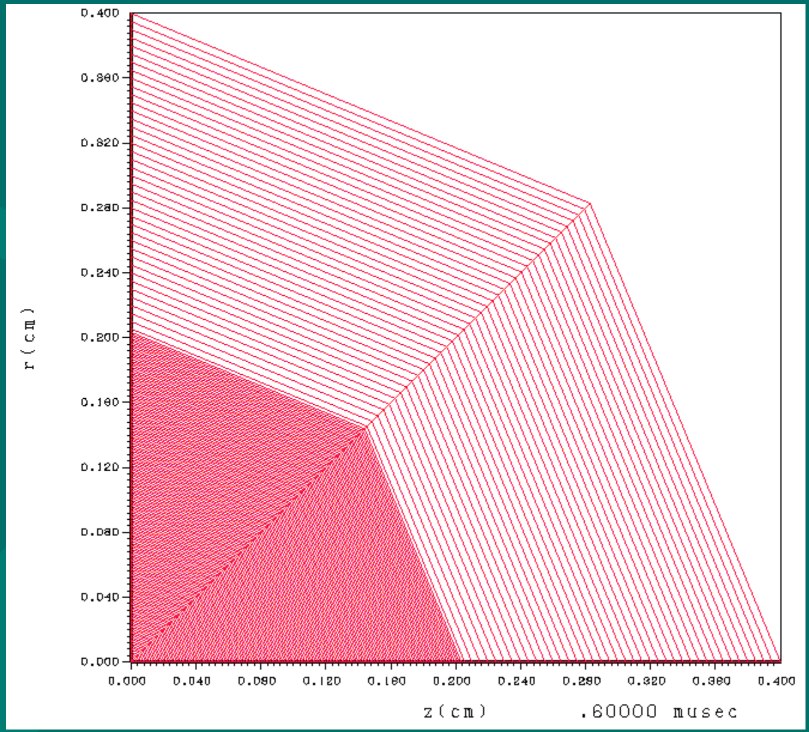
- Corner masses corresponding to these volumes are calculated and stored at the start of a calculation.
- The volumes and areas are recalculated every time step and the density required to match the corner mass with the new corner volume is used to define the area weighted masses used in the momentum step.
- Since the nodes on axis have zero mass the area weighted corner masses are set for these nodes using the average cell density of the nearest nodes off axis or the motion of the nodes is simply inferred from the nearest node off axis.

Edge q's

- Edge q's have been implemented largely following Caramana et al's paper, but collapsing to a scalar rather than a vector.
- The force contribution for each edge q has been calculated by defining a triangle with vertices again defined by ends of edge and the cell center.
- Each triangle was then treated as a degenerate quad to allow forces to be calculated using the existing Finite Element Method in CORVUS.
- The forces at the cell centre were treated as non-dynamical points and split equally between the two edge nodes.

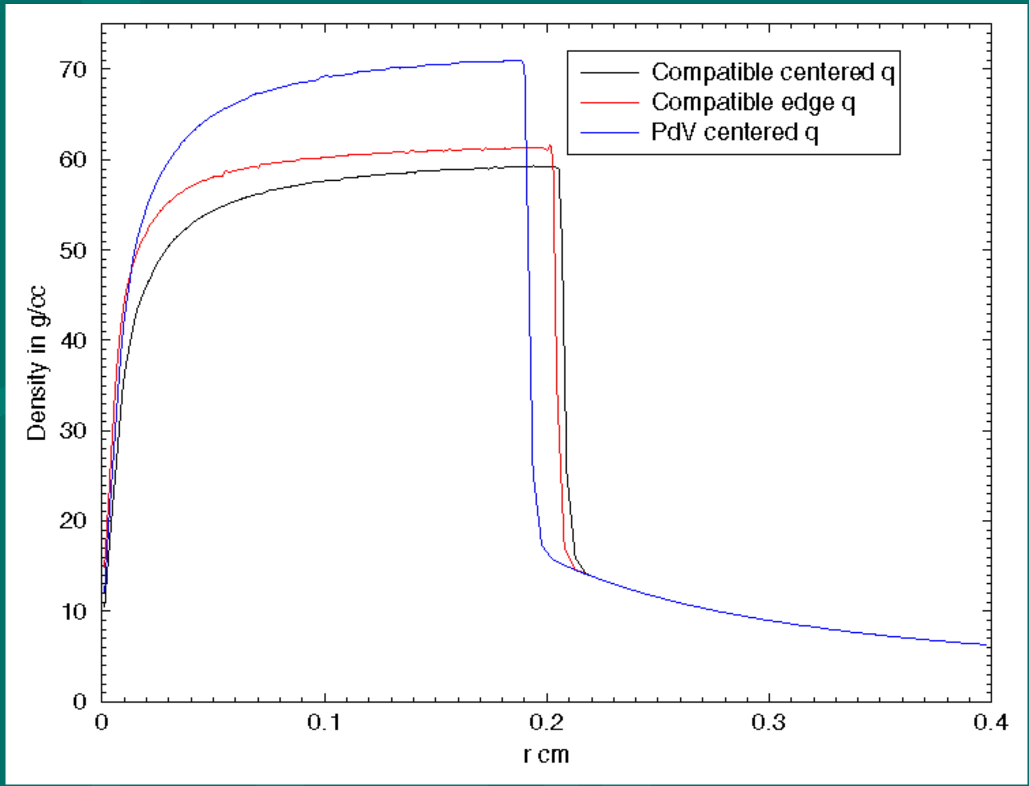
Noh Problem 1





- Standard PdV scheme in CORVUS has lost 4% of its total energy by 0.6 μs for 2x200 Noh Problem.
- Compatible Hydro conserves total energy to round of for element centered or edge monotonic q.

Noh Problem 2



- Energy loss with PdV scheme leads to density exceeding analytical solution behind shock (64 g/cc).
- Compatible hydro appears to be converging to analytic solution with edge q converging faster than cell centered monotonic q.

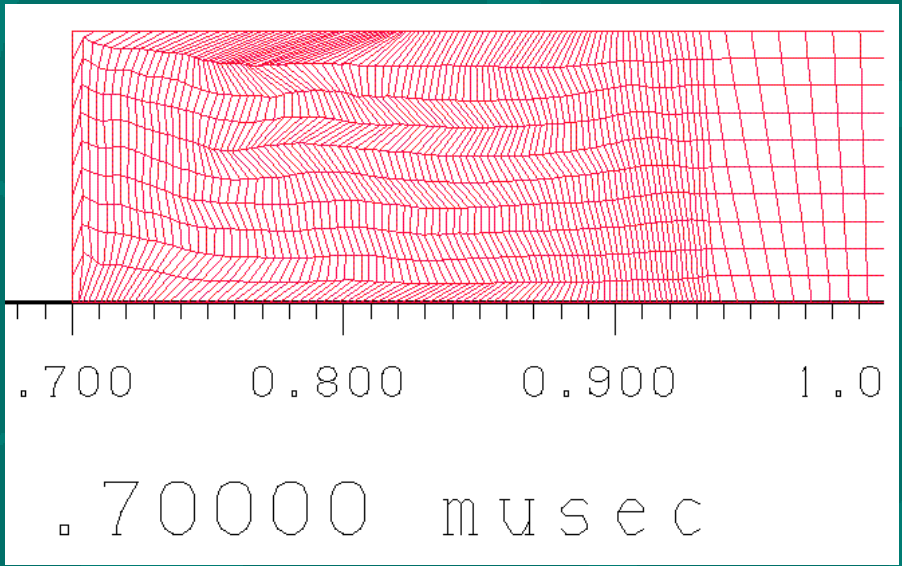
Sub-Zonal pressures 1

- Sub-zonal pressures have been implemented so they can be compared directly against the existing hourglass filters in CORVUS.
- Sub-zonal pressures have been implemented by treating each element as being subdivided into four sub elements defined by the 4 existing vertices, cell center and 4 half side points.
- The initial mass for these 4 volumes is stored for each cell and used each time step to define sub-zonal densities.
- Assuming the internal energy is constant across the zone subzonal pressures can then be calculated.

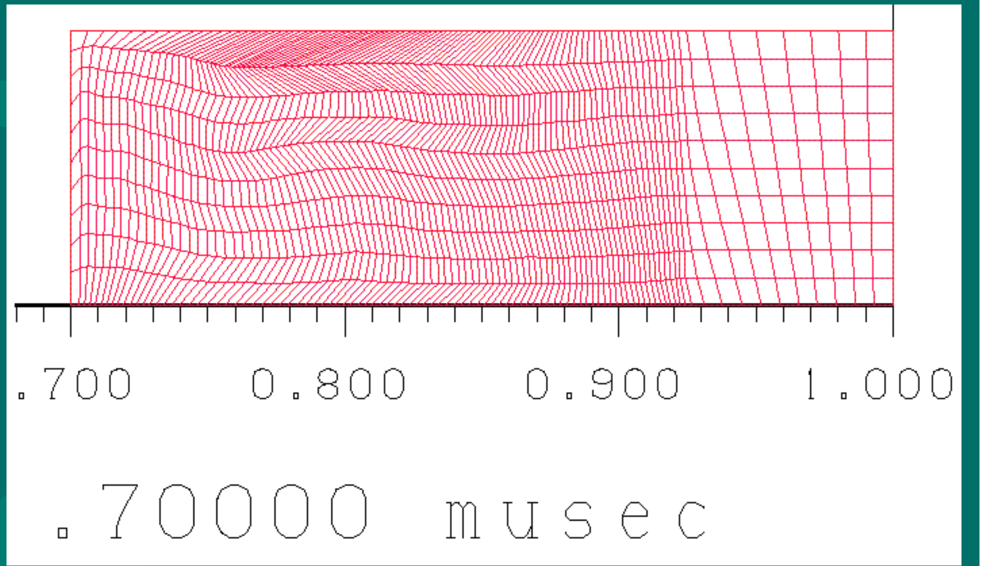
Sub-Zonal pressures 2

- The subzonal pressures are then treated as perturbations to the standard element pressure.
- Associated forces are then calculated using the usual Finite Element approach for the four subzones.
- The center point and half side points are then treated as non-dynamical points with their associated forces being distributed to the appropriate dynamical points.
- The subzone pressure perturbation forces are then added to the normal pressure forces for the zone.

Saltzman's Piston 1

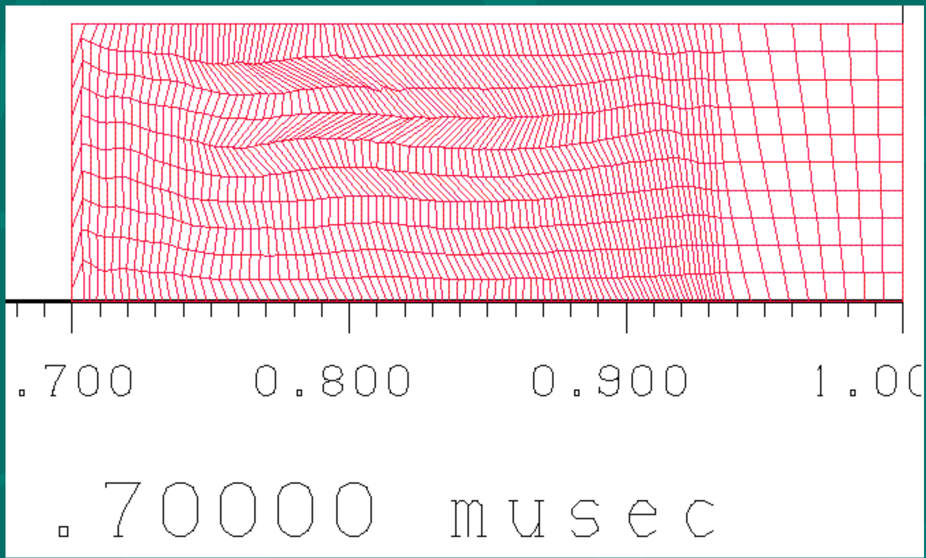


PdV Element centred scalar monotonic q

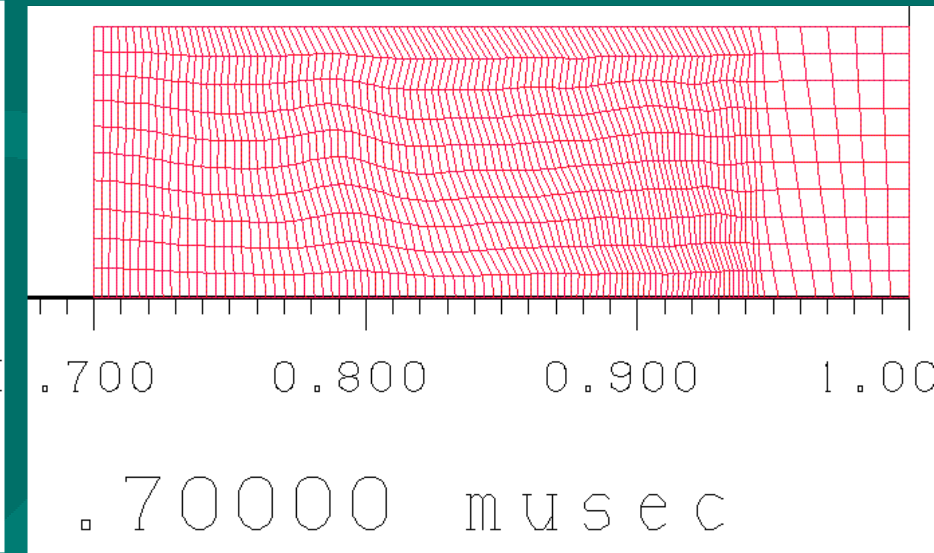


PdV Element centred scalar monotonic q with default hourglass filter (hgtype=3, kappa=0.01)

Saltzman's Piston 2

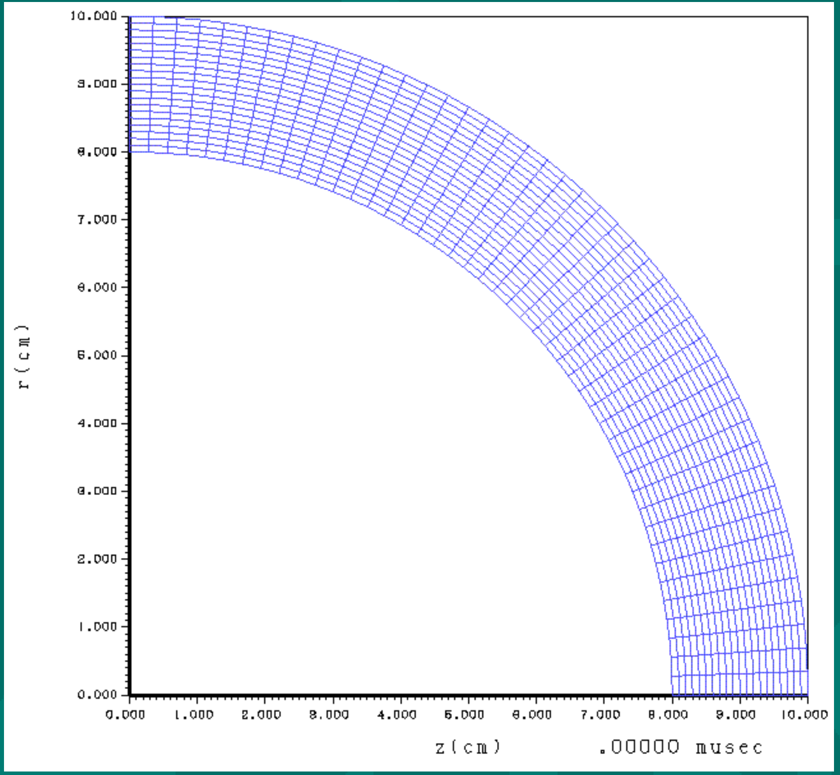


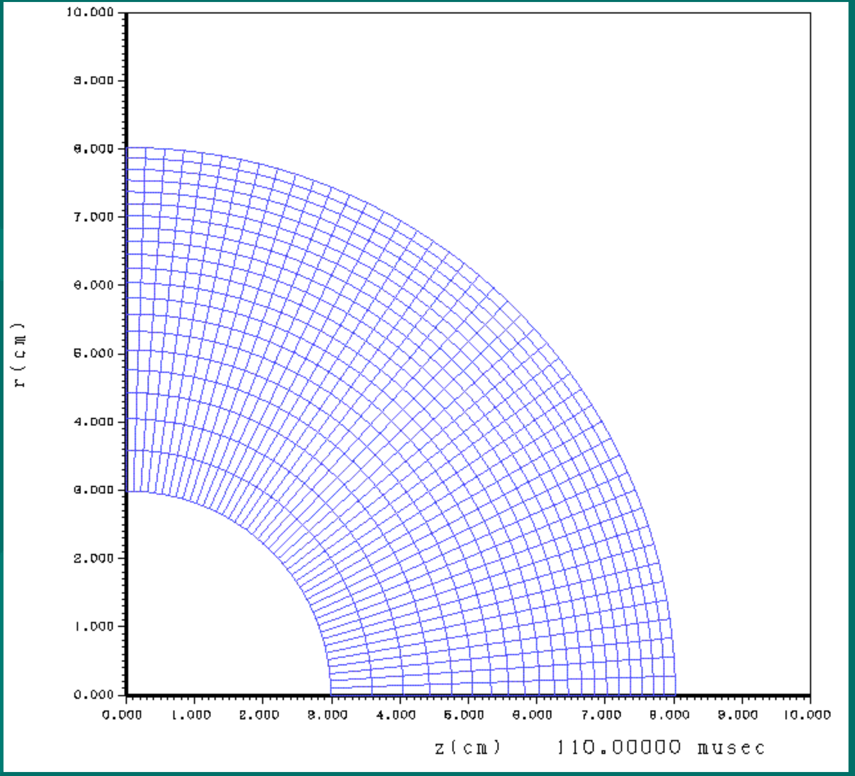
Compatible Hydro with Element centred scalar monotonic q



Compatible Hydro with Element centred scalar monotonic q with subzone pressures using merit factor of 1

Be Stopping Shell





- Be stopping shell problem is a material strength test which tests the conversion of kinetic energy into plastic work for a nearly incompressible problem.
- PdV scheme looses 0.05%.
- Compatible hydro conserves energy exactly to round off. Close agreement to analytical solution for stopping radius (3 cm).

Compatible Slide (1)

- CORVUS employs a master/slave algorithm. This has been modified to exploit the benefits of the compatible hydro.
  - Free surface accelerations are calculated for all slide nodes.
  - Normal accelerations are modified for all slide nodes to allow for the interaction with the other surface.
  - Positions and velocities for slide nodes are then updated to reflect the modified accelerations.

Compatible Slide (2)

- Slave nodes are then put back on to master by finding intersection with unit normal for slave node as before.
- Normal velocities for slave nodes interpolated from master surface. **Change in kinetic energy for corner masses of the slave slide nodes are then calculated and used to modify internal energy of elements on slave slide line to conserve total energy.**
- **Corrector internal energy work update is performed using average nodal velocities to reach position before slave nodes are put back on to master.**

Original Void Closure Scheme

- The original void closure scheme in CORVUS was implemented as a modified putback on step.
- A slide node is treated as closed if the slaves normal projected back into the material intersects with the master surface.
- The normal velocity of the slave nodes that were flagged as closed were then interpolated from the master surface and rest of the slide step performed unmodified.

Compatible Void closure (1)

- In practice the original CORVUS void closure scheme showed significant differences depending on which way the master and slave were chosen and did not conserve energy well enough.
- This appears to be mainly due to interpolating the slave nodes velocity from the master.
- A new compatible scheme has been developed which modifies the normal velocities of the master and slave nodes on contact.

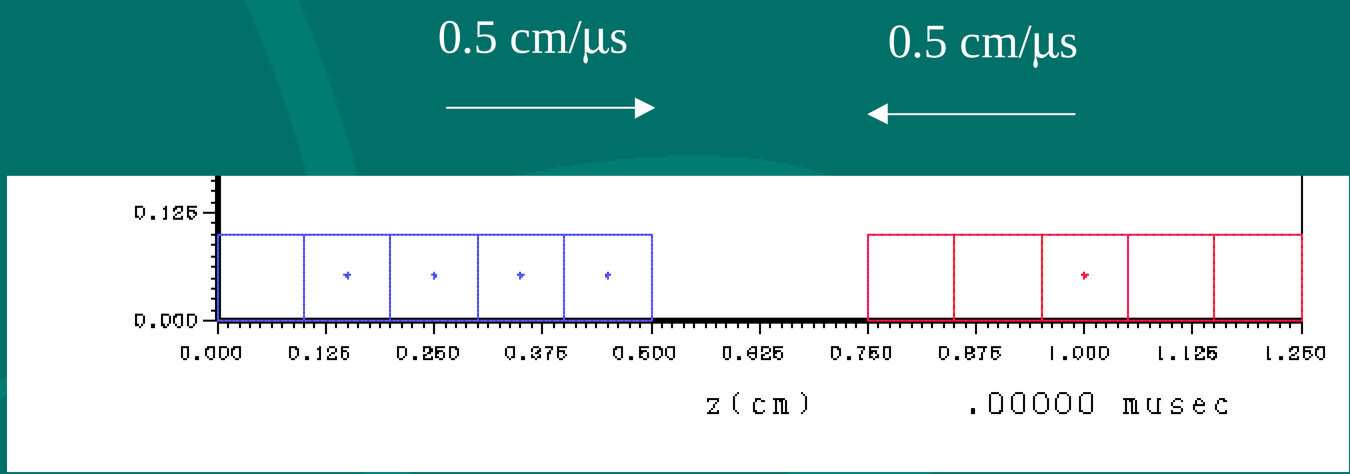
Compatible Void closure (2)

- The normal velocities for both surfaces are obtained by applying the conservation of momentum normal to the interface.
- This is achieved by considering the impact of each real node with a pseudo node on the opposite slide surface.
- The mass and normal velocity of each pseudo node is interpolated from its immediate real neighbours.

Compatible Void closure (3)

- The change in kinetic energy for corner masses of the master and slave slide nodes are then calculated and used to modify internal energy of elements on both the master and slave slide lines to conserve total energy.
- The approach could be applied as a replacement for the normal velocity interpolation for the standard slide as well. However, in practice it is is less robust than the normal velocity interpolation and can lead to perturbation along slide lines.
- So is only applied as a void is closed where the greatest benefits are obtained.

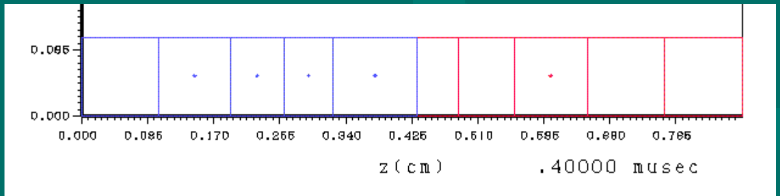
1D Impact test problem

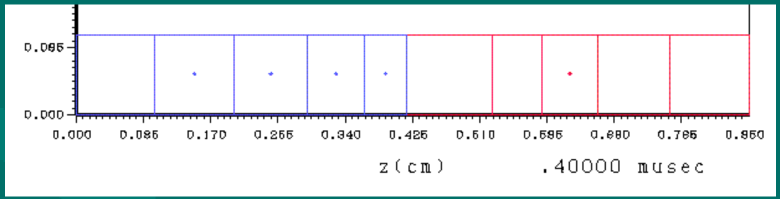


- In order to test the symmetry of the void closure scheme a simple test problem has been performed involving the impact of two identical Tantalum rods both initially travelling at 0.5 cm/µs towards each other.

1D Impact test problem

Original CORVUS Scheme

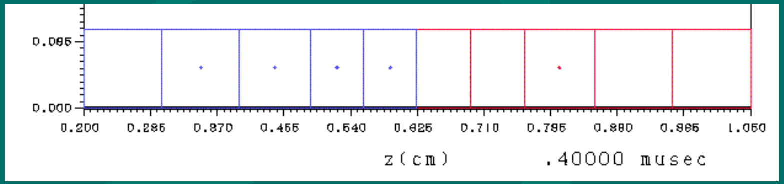


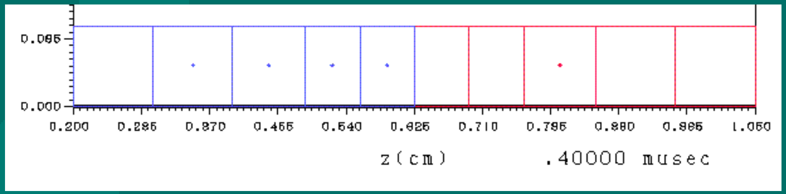


Master on right

T=0.4µs

Master on left



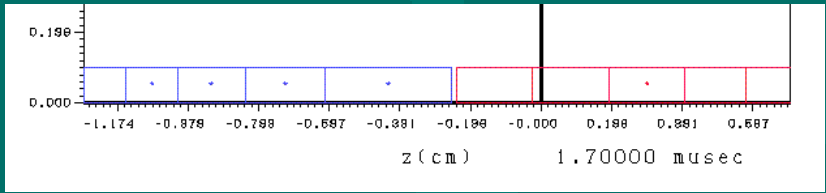


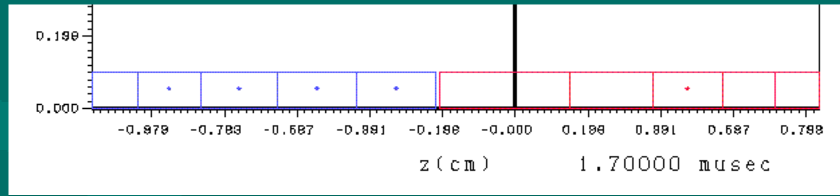
New Compatible Scheme

- 1D impact test problem shows sensitivity to choice of master and slave with the original CORVUS scheme.
- While the new compatible scheme produces symmetric results which ever way the master and slave are ordered.

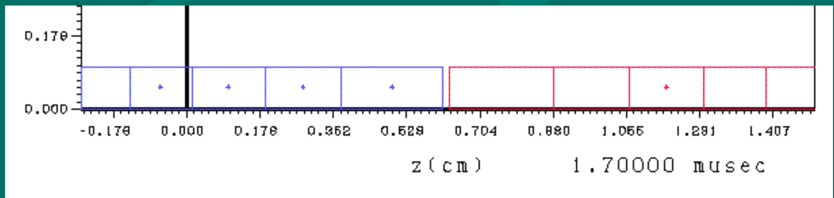
1D Impact test problem

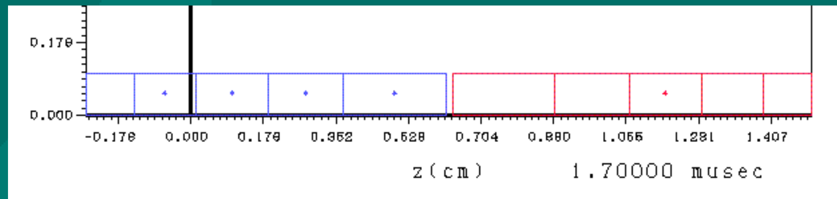
Original CORVUS Scheme





Master on right T=1.7 μs Master on left

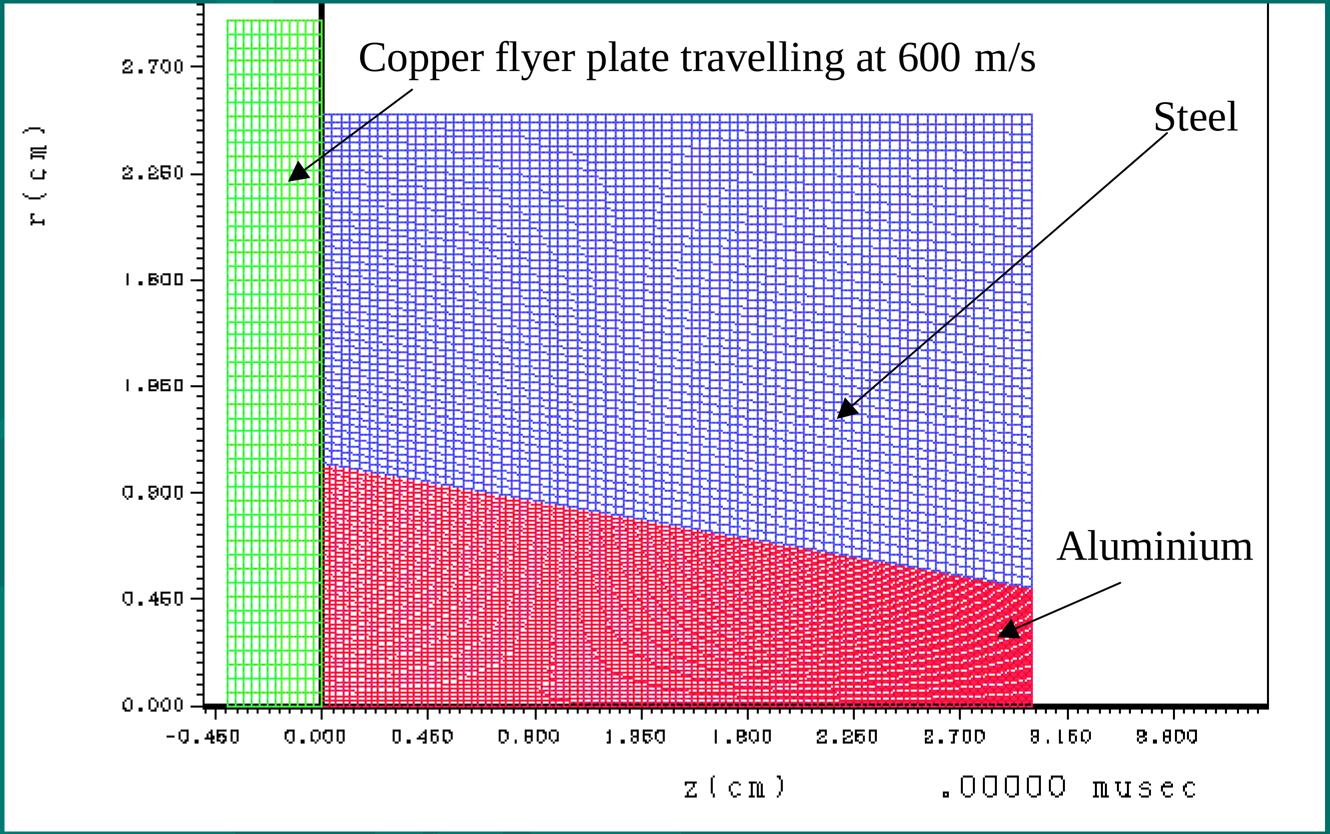




New Compatible Scheme

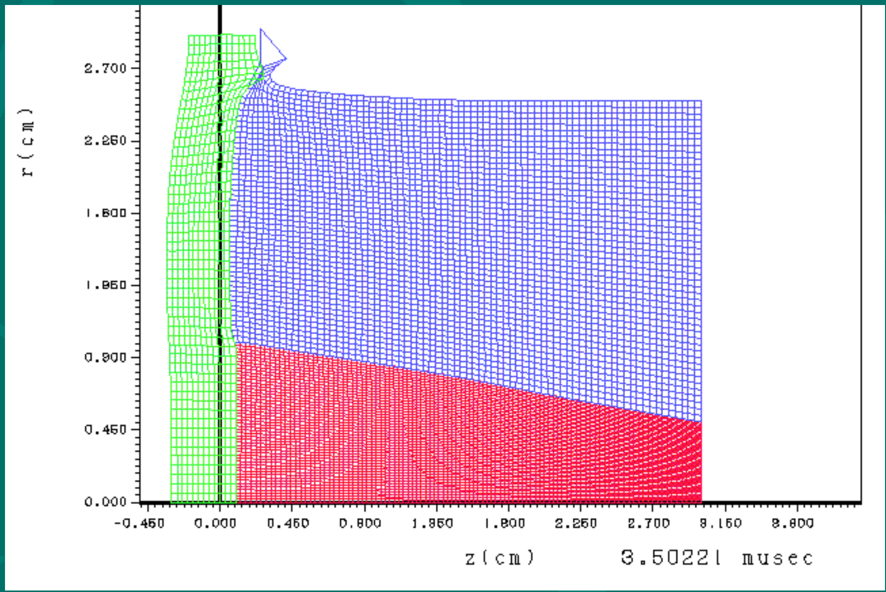
- Sensitivity to the choice of master and slave persists with original scheme and effects the position where the plates separate. Kinetic energy is also not distributed equally between the two plates.
- In contrast with the new scheme the plates separate at the same position and the kinetic energy is shared almost equally between the two which ever way the master and slave are ordered.

2D Impact and Slide Test

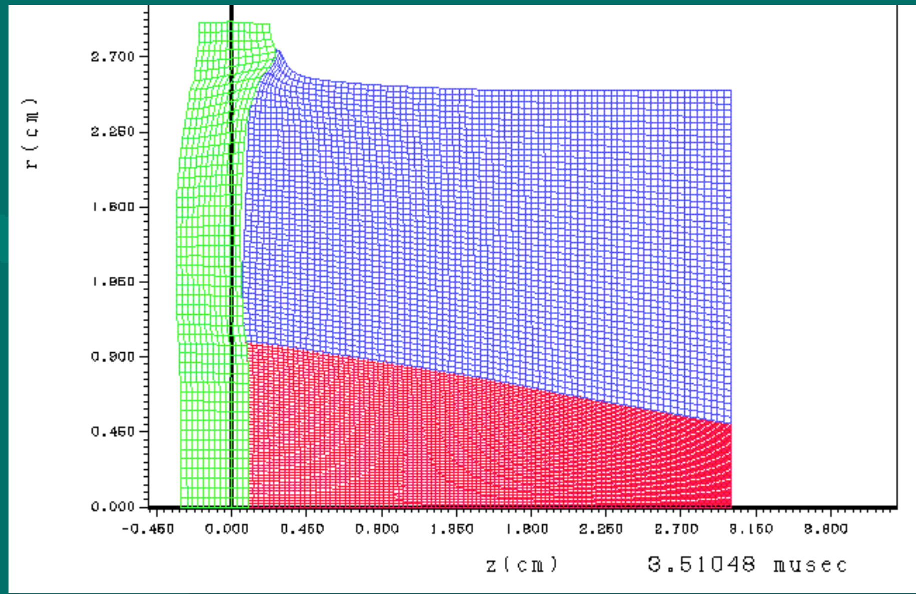


- A friction experiment driven by a flyer plate has also been tested as it provides a more realistic test of both slide and void closure.

2D Impact and Slide Test



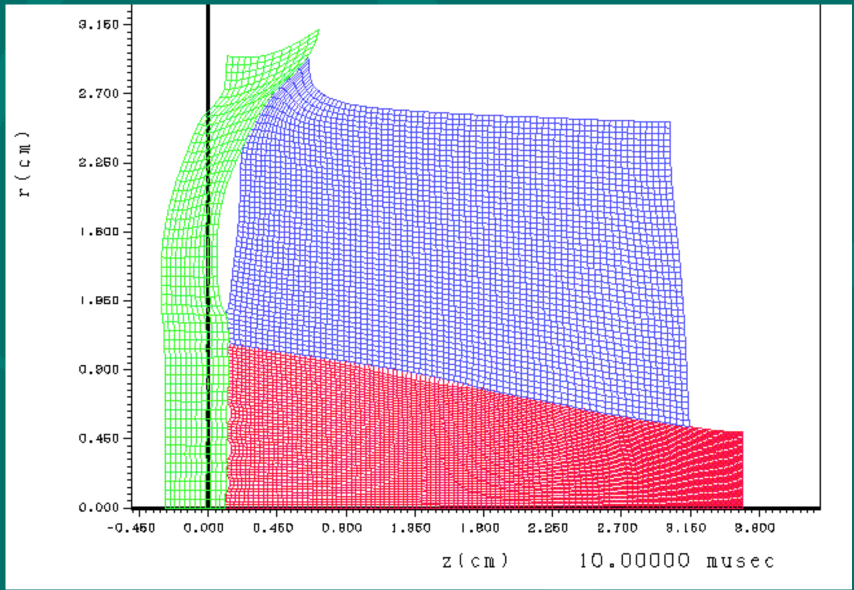
Original CORVUS hydro scheme



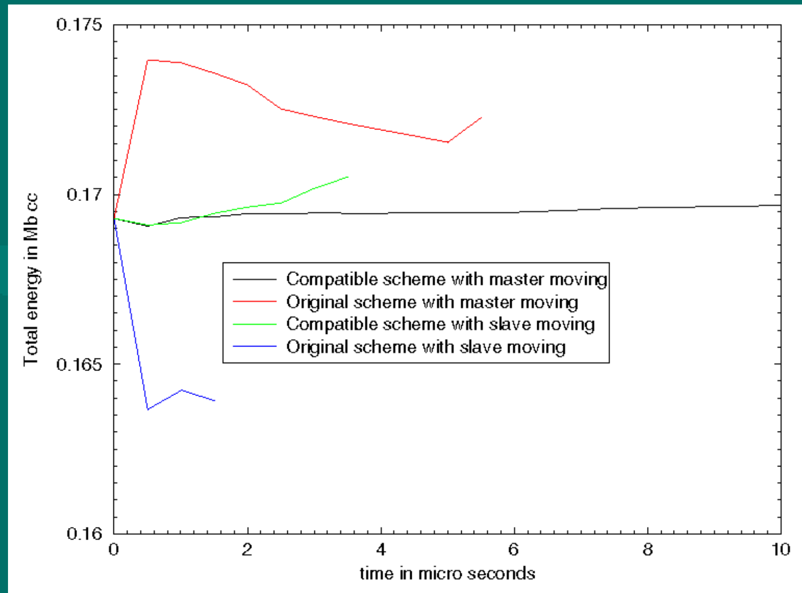
Compatible hydro

- Original CORVUS hydro scheme does not complete running pure Lagrangian due to mesh tangling at the end of the slide line.
- Lagrangian calculation with compatible hydro and sub-zonal pressure scheme runs robustly to completion.

2D Impact and Slide Test



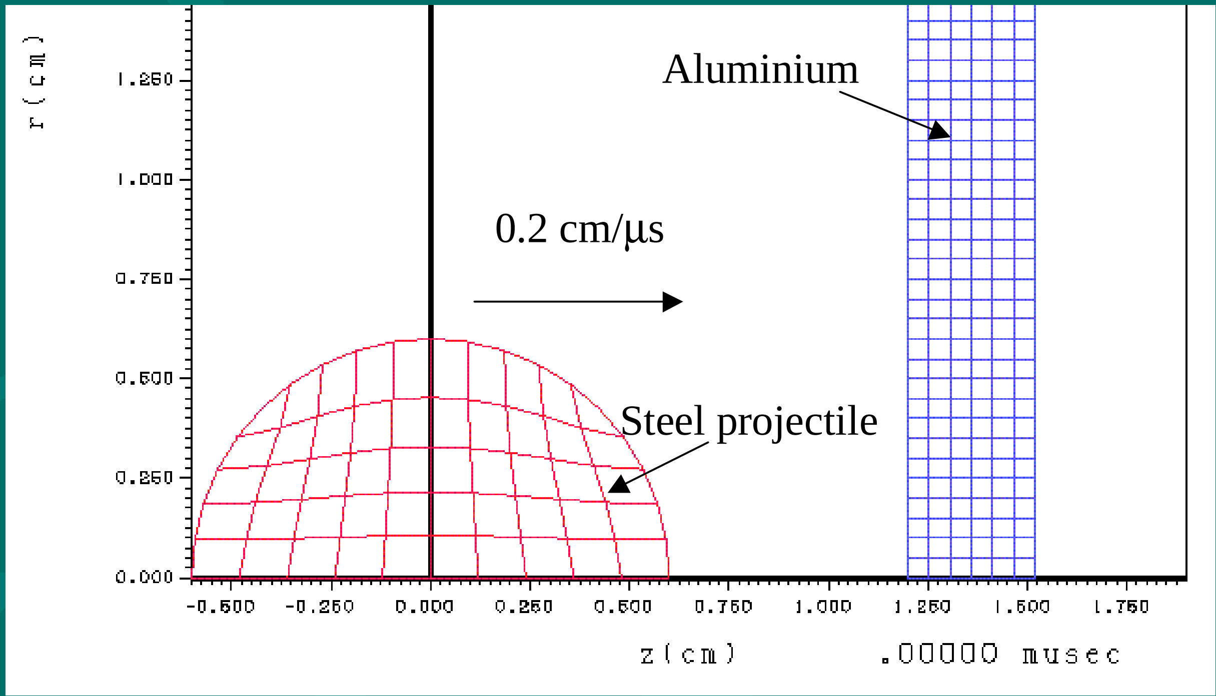
Compatible hydro at t=10.0µs



Total energy v time for new and old hydro schemes

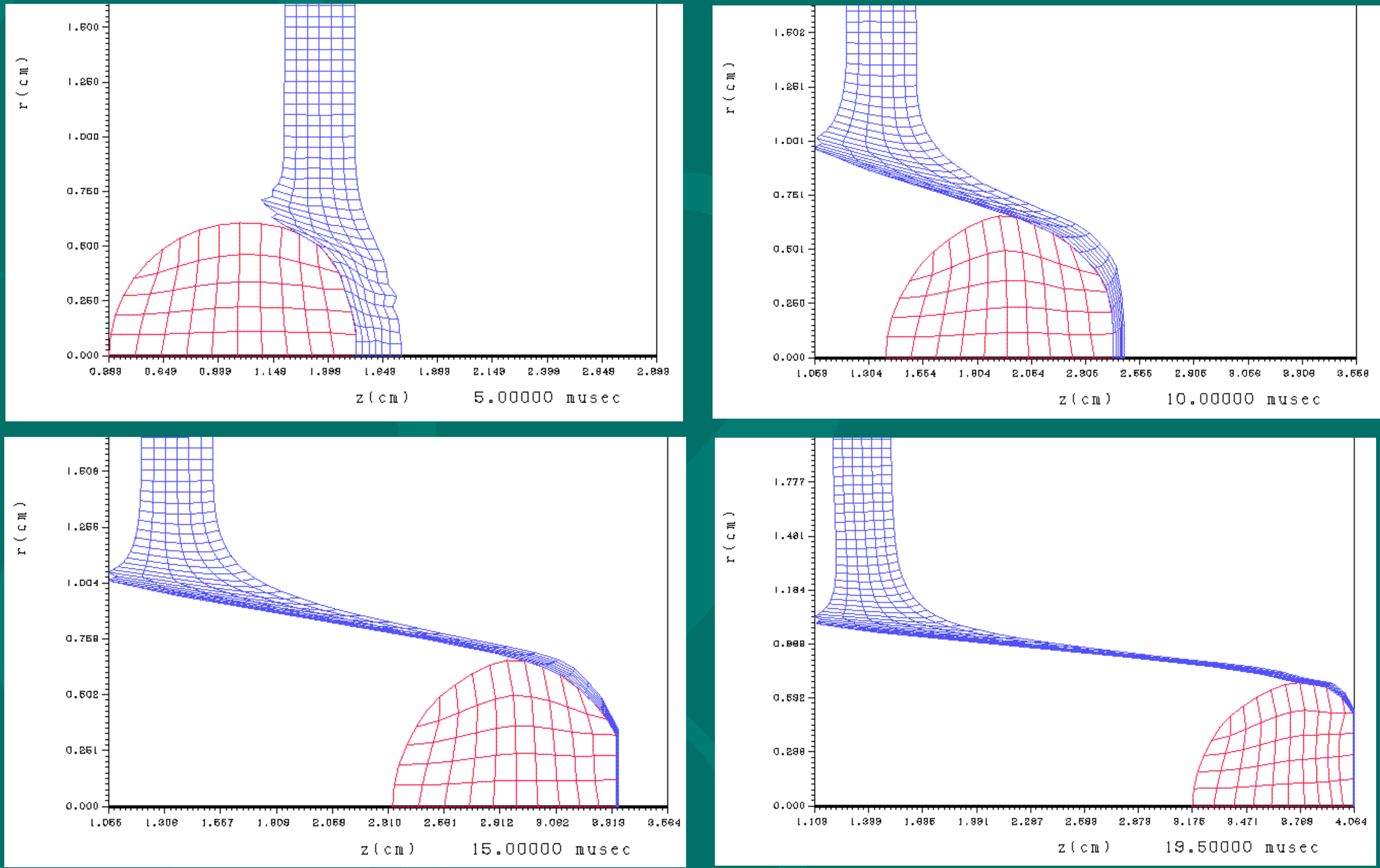
- The new compatible slide scheme conserves energy significantly better than the original CORVUS scheme and shows far less sensitivity to the choice of master and slave.
- Void closure shows the greatest improvement, but standard slide is also improved.

Projectile impact problem

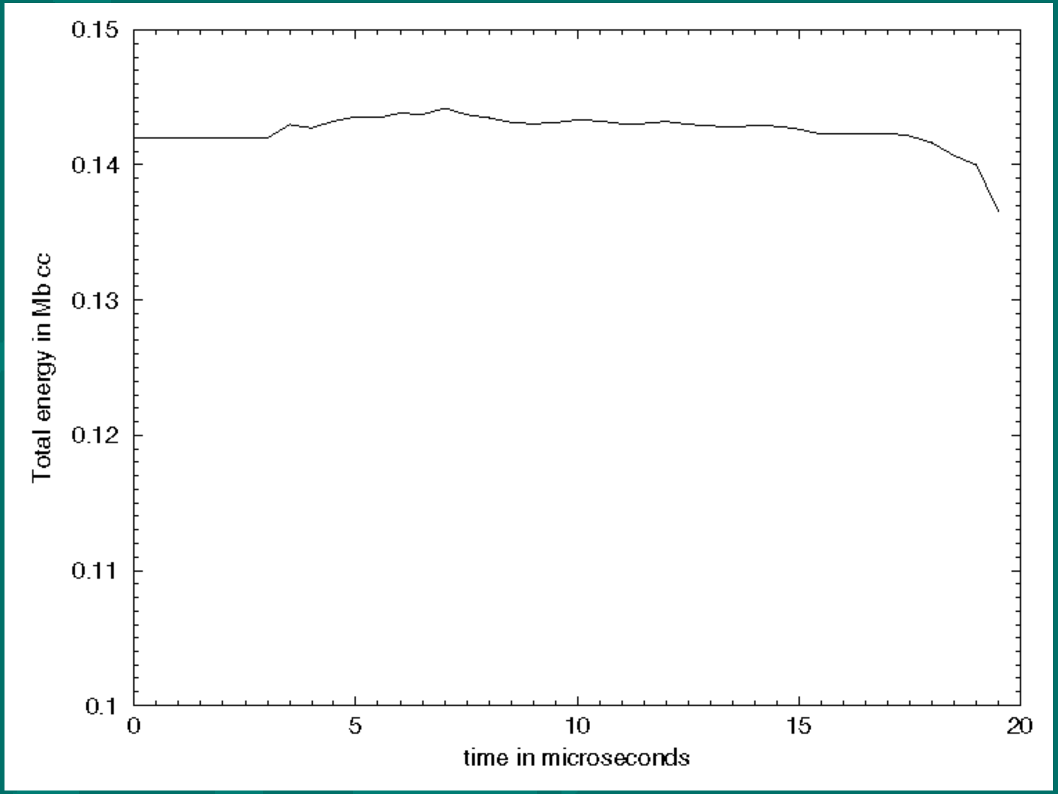


- A projectile impact problem has also been calculated to provide a more demanding test of the robustness and energy conservation of the new slide and void closure scheme.

Projectile impact problem



Projectile impact problem



- The new compatible slide and void closure scheme enables the projectile impact problem to be run robustly with Lagrangian mesh motion to significantly later times than was possible with the original CORVUS scheme and conserves energy.

Conclusion

- A new compatible finite element Lagrangian hydro scheme has been developed for CORVUS.
- This has demonstrated that the main ideas of compatible finite volume hydro can be transferred across to finite element methods.
- The slide and void closure algorithms in CORVUS have also been improved by exploiting the ideas of compatible hydro and improving the symmetry of the put back on step for void closure.
- The new finite element compatible hydro scheme has also been shown to offer real benefits over the original PdV based scheme in CORVUS for a number of real applications.

Future Work

- Make consistent with Friction and ALE.
- Make fully consistent with radiation PdV.
- Continue to assess potential benefits for applications.